

Radiative Electroweak Symmetry Breaking and the Infrared Fixed Point of the Top Quark Mass

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Abstract

The infrared quasi fixed point solution for the top quark mass in the Minimal Supersymmetric Standard Model explains in a natural way large values of the top quark mass and appears as a prediction in many interesting theoretical schemes. Moreover, as has been recently pointed out, for moderate values of $\tan\beta$, in order to achieve gauge and bottom-tau Yukawa coupling unification, the top quark mass must be within 10% of its fixed point value. In this work we show that the convergence of the top quark mass to its fixed point value has relevant consequences for the (assumed) universal soft supersymmetry breaking parameters at the grand unification scale. In particular, we show that the low energy parameters do not depend on A_0 and B_0 but on the combination $\delta = B_0 - A_0/2$. Hence, there is a reduction in the number of independent parameters. Most interesting, the radiative $SU(2)_L \times U(1)_Y$ breaking condition implies strong correlations between the supersymmetric mass parameter μ and the supersymmetry breaking parameters δ and $M_{1/2}$ or m_0 . These correlations, which become stronger for $\tan\beta < 2$, may have some fundamental origin, which would imply the need of a reformulation of the naive fine tuning criteria.

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1 Introduction

The increasing lower bound on the top quark mass has led to a renewed interest in the fixed point solutions for the top quark Yukawa coupling [1]-[3]. In particular, the infrared quasi fixed point for the top quark mass in the Minimal Supersymmetric Standard Model [4] appears naturally in the framework of models with dynamical breaking of the electroweak symmetry, the so-called top condensate models. This is due to the fact that this solution is associated with the renormalization group trajectories on which the top quark Yukawa coupling, h_t , becomes large, $Y_t \equiv h_t^2/4\pi = \mathcal{O}(1)$, at scales of order 10^{16} GeV. Most interesting, it has recently been pointed out that, for moderate values of the ratio of Higgs vacuum expectation values, the condition of bottom-tau Yukawa coupling unification in minimal supersymmetric grand unified theories [5]-[7], requires large values for the top quark Yukawa coupling at the grand unification scale. This behaviour arises from the necessity of contravening the strong gauge coupling renormalization effects on the bottom Yukawa coupling [8]-[10]. For the values of the gauge couplings allowed by the most recent experiments at LEP and from the grand unification condition, it follows that the top quark mass required to achieve bottom-tau Yukawa coupling unification must be within 10% of its infrared quasi fixed point value [11].

The above predictions for the top quark mass are independent of the source of the soft supersymmetry breaking mass parameters. In fact, since the only strong dependence of the infrared quasi fixed point prediction on the supersymmetric spectrum comes through the strong gauge coupling α_3 , this dependence can be characterized, in grand unification scenarios, by an effective supersymmetric threshold scale T_{SUSY} [9], [10], which defines the value of $\alpha_3(M_Z)$ for a fixed value of the weak gauge couplings. The fixed point value for the top quark mass is given by $M_t = C \sin \beta$, with $C \simeq 190 - 210$ GeV for $\alpha_3(M_Z) = 0.11 - 0.13$ and $\tan \beta$ being the ratio of the vacuum expectation values of the Higgs fields. For instance, a top quark mass $M_t \leq 180$ GeV may only be obtained for $\tan \beta \leq 2$ (or for very large values of $\tan \beta$).

In the present work we shall analyse the potential implications for the minimal supergravity model of the top quark mass being at its infrared quasi fixed point value. In this model, the low energy soft supersymmetry breaking mass parameters are thought to proceed from common given values at the grand unification scale, and the electroweak symmetry is broken radiatively. As we shall show below, for low and moderate values of $\tan \beta$, the evolution of the soft supersymmetry breaking parameters may be given as a function of the ratio of the top quark Yukawa coupling Y_t to its fixed point value Y_f ,

yielding definite analytical predictions in the limit $Y_t \rightarrow Y_f$.

In addition, since the low energy parameters must give a proper breakdown of the $SU(2)_L \times U(1)_Y$ symmetry, there is usually some degree of fine tuning, which is increased for low $\tan\beta \simeq 1$ as well as for very large values of $\tan\beta$. However, we shall show that, when the radiative breaking condition on the supersymmetry breaking parameters is imposed, it leads to relevant correlations between the different high energy parameters. These correlations are stronger for exactly those values of $\tan\beta$ for which the naive fine tuning is strong. They may have some fundamental explanation, which would make the usual fine tuning argument inappropriate. This result applies in particular to the region $\tan\beta \leq 2$, which corresponds to the infrared quasi fixed point values of the top quark mass $M_t \leq 180$ GeV.

In the following, we shall perform a detailed analysis of the properties mentioned above in the region of small and moderate values of $\tan\beta$. The large $\tan\beta$ region ($\tan\beta > 30$) will be analysed in a forthcoming paper. In our present study we shall use the bottom - up approach introduced in Ref. [12], which enables a clear formulation of these properties: while scanning the whole low energy region in our search for correlations of the mass parameters at M_{GUT} we can define the exact patterns required to be close to the infrared quasi fixed point. We shall also make use of analytical solutions for the low energy parameters, which are extremely useful in understanding the properties derived from the numerical study. In section 2 we give analytical one loop expressions, which show the dependence of the low energy scalar mass parameters on the high energy soft SUSY breaking mass parameters and on the top quark Yukawa coupling. In section 3 we analyse the implications of the infrared fixed point solution for the scalar mass parameter evolution. In section 4 we incorporate the radiative breakdown of the electroweak symmetry and derive approximate analytical relations between the soft supersymmetry breaking parameters at the high energy scale. We compare our analytical results with those we obtain from the full numerical computations, and we search for correlations between the different high energy parameters. We reserve section 5 for our conclusions.

2 Higgs Potential Parameters

The Higgs potential of the Minimal Supersymmetric Standard Model may be written as [4], [13]-[15]

$$V_{eff} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 (H_1^T i\tau_2 H_2 + h.c.)$$

$$+ \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 |H_2^\dagger i\tau_2 H_1^*|^2, \quad (1)$$

where at scales at which the theory is supersymmetric the running quartic couplings λ_j , with $j = 1 - 4$, must satisfy the following conditions:

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \quad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \quad \lambda_4 = -\frac{g_2^2}{2}. \quad (2)$$

Hence, in order to obtain the low energy values of the quartic couplings, they must be evolved using the appropriate renormalization group equations, as was explained in Refs. [13]-[16]. The mass parameters m_i^2 , with $i = 1-3$ must also be evolved in a consistent way below the supersymmetry breaking scale. The minimization conditions read

$$\sin(2\beta) = \frac{2m_3^2}{m_A^2} \quad (3)$$

$$\tan^2 \beta = \frac{m_1^2 + \lambda_2 v^2 + (\lambda_1 - \lambda_2) v_1^2}{m_2^2 + \lambda_2 v^2}, \quad (4)$$

where $\tan \beta = v_2/v_1$, v_i is the vacuum expectation value of the Higgs fields H_i , $v^2 = v_1^2 + v_2^2$, m_A is the CP-odd Higgs mass,

$$m_A^2 = m_1^2 + m_2^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4) v^2 \quad (5)$$

and we define the mass parameter m_3^2 to be positive.

The renormalization group equations for the mass parameters may be found in the literature [17]-[20]. Apart from the mass parameters m_i^2 , appearing in the effective potential, the evolution of the supersymmetric mass parameter μ appearing in the superpotential f ,

$$f = h_t \epsilon_{ab} Q^b U H_2^a + \mu \epsilon_{ab} H_1^a H_2^b, \quad (6)$$

is also relevant for the analysis of the radiative electroweak symmetry breaking conditions. In the above, $Q^T = (T \ B)$ is the bottom-top left handed doublet superfield and $U \equiv T^C$. In Eq. (6) we have just written the top quark Yukawa contribution, which is the only one relevant for our analysis, since we are restricting it to the region in which $\tan \beta$ takes small or moderate values. The bilinear mass term proportional to m_3^2 appearing in the Higgs potential may be rewritten as a soft supersymmetry breaking parameter B multiplied by the Higgs bilinear term appearing in the superpotential, that is $m_3^2 = B\mu$. Analogously, the full scalar potential contains scalar trilinear supersymmetry breaking terms with couplings A_f , proportional to the terms in the superpotential associated with the Yukawa couplings h_f .

The minimal supergravity model is obtained by assuming the universality of the soft supersymmetry breaking parameters at the grand unification scale: common soft supersymmetry breaking mass terms m_0 and $M_{1/2}$ for the scalar and gaugino sectors of the theory, respectively, and a common value A_0 for all trilinear couplings A_f . At the grand unification scale, the mass parameters B and μ take values B_0 and μ_0 , respectively. Knowing the values of the mass parameters at the unification scale, their low energy values may be specified by their renormalization group evolution. In the region of small and moderate values of $\tan \beta$, for which the bottom and tau Yukawa coupling effects may be safely neglected, an analytical solution for the evolution of the mass parameters may be obtained, for any given value of the top quark Yukawa coupling.

The solution for the top quark Yukawa coupling, in terms of Y_t , reads [19],[20]:

$$Y_t(t) = \frac{2\pi Y_t(0)E(t)}{2\pi + 3Y_t(0)F(t)}, \quad (7)$$

with E and F being functions of the gauge couplings,

$$E = (1 + \beta_3 t)^{16/3b_3} (1 + \beta_2 t)^{3/3b_2} (1 + \beta_1 t)^{13/9b_1}, \quad F = \int_0^t E(t') dt', \quad (8)$$

where $\beta_i = \alpha_i(0)b_i/4\pi$, b_i is the beta function coefficient of the gauge coupling α_i and $t = 2\log(M_{GUT}/Q)$. As we mentioned above, the fixed point solution is obtained for values of the top quark Yukawa coupling that become large at the grand unification scale, that is, approximately

$$Y_f(t) = \frac{2\pi E(t)}{3F(t)}. \quad (9)$$

From here, considering the renormalization group evolution of the mass parameters [19]-[21], the following approximate analytical solutions are obtained,

$$m_{H_1}^2 = m_0^2 + 0.5M_{1/2}^2, \quad m_{H_2}^2 = m_{H_1}^2 + \Delta m^2, \quad (10)$$

where $m_i^2 = \mu^2 + m_{H_i}^2$, with $i = 1, 2$, and

$$\begin{aligned} \Delta m^2 = & -\frac{3m_0^2}{2} \frac{Y_t}{Y_f} + 2.3A_0 M_{1/2} \frac{Y_t}{Y_f} \left(1 - \frac{Y_t}{Y_f}\right) \\ & - \frac{A_0^2}{2} \frac{Y_t}{Y_f} \left(1 - \frac{Y_t}{Y_f}\right) + M_{1/2}^2 \left[-7 \frac{Y_t}{Y_f} + 3 \left(\frac{Y_t}{Y_f}\right)^2\right]. \end{aligned} \quad (11)$$

Moreover, the renormalization group evolution for the supersymmetric mass parameter μ reads,

$$\mu^2 = 2\mu_0^2 \left(1 - \frac{Y_t}{Y_f}\right)^{1/2}, \quad (12)$$

while the running of the soft supersymmetry breaking bilinear coupling gives,

$$B = B_0 - \frac{A_0}{2} \frac{Y_t}{Y_f} + M_{1/2} \left(1.2 \frac{Y_t}{Y_f} - 0.6 \right). \quad (13)$$

Finally, it is also useful to present the evolution of the supersymmetry breaking mass parameters of the supersymmetric partners of the left and right handed top quarks,

$$m_Q^2 = 7.2M_{1/2}^2 + m_0^2 + \frac{\Delta m^2}{3} \quad , \quad m_U^2 = 6.7M_{1/2}^2 + m_0^2 + 2\frac{\Delta m^2}{3}, \quad (14)$$

respectively. We shall concentrate on the renormalization group evolution of the supersymmetric mass parameters given above, Eqs. (10) - (14), since within the bottom - up approach introduced in Ref. [12] these are sufficient for the determination of the high energy parameters.

A remark is in order. The above solutions have been obtained by using the perturbative one loop renormalization group equations for the gauge and Yukawa couplings, as well as for the mass parameters. Hence, they can only be used for values of the top quark Yukawa coupling at the grand unification scale within the range of validity of perturbation theory, $Y_t(0) \leq 1$. Since there is a one to one relationship between the values of $Y_t(0)$ and the degree of convergence of the top quark Yukawa coupling to its infrared quasi fixed point value, this bound implies that the solutions associated with top quark Yukawa couplings that are closer than 0.5% to the fixed point value cannot be studied within the one loop approximation. In the following, when talking about the limit $Y_t \rightarrow Y_f$, we will be implicitly assuming that we are working within the range of validity of perturbation theory. In addition, for values of Y_t that are very close to its fixed point value, two loop effects may become important. Therefore, in our numerical solution we have considered the full two loop renormalization group evolution for gauge and Yukawa couplings.

The coefficients characterizing the dependence of the mass parameters on the universal gaugino mass $M_{1/2}$ depend on the exact value of the gauge couplings. In the above, we have taken the values of the coefficients that are obtained for $\alpha_3(M_Z) \simeq 0.12$. The above analytical solutions are sufficiently accurate for the purpose of understanding the properties of the mass parameters in the limit $Y_t \rightarrow Y_f$. We shall then confront the results of our analytical study with those obtained from the numerical two loop analysis.

3 Properties of the Fixed Point Solutions

The above expressions show important properties of the solution when $Y_t \rightarrow Y_f$:

a) The mass parameters $m_{H_2}^2$, m_Q^2 and m_U^2 become very weakly dependent on the supersymmetry breaking parameter A_0 . In fact, the dependence on A_0 vanishes in the formal limit $Y_t \rightarrow Y_f$. The only relevant dependence on A_0 enters through the mass parameter m_3^2 . This leads to property (b).

b) There is an effective reduction in the number of free independent soft supersymmetry breaking parameters. In fact, the dependence on B_0 and A_0 of the low energy solutions is effectively replaced by a dependence on the parameter

$$\delta = B_0 - \frac{A_0}{2}. \quad (15)$$

c) From Eq. (12), it follows that the coefficient relating μ to μ_0 tends to zero as $Y_t \rightarrow Y_f$. This tendency is, however, much slower than that of the coefficient associated with the A_0 dependence of the mass parameters leading to properties (a) and (b). For instance, even if Y_t lies as close as only 0.5% away from Y_f , this coefficient is still of order one. The relevant property following from Eq. (12) is that, for the same low energy value of μ , consistent with the radiative breaking of $SU(2)_L \times U(1)_Y$, μ_0 should scale like $\left(1 - \frac{Y_t}{Y_f}\right)^{-1/4}$.

d) There is a very interesting dependence of the low energy mass parameters on m_0 . For example, the m_0 dependence of the combination $m_Q^2 + m_{H_2}^2$ vanishes in the formal limit $Y_t \rightarrow Y_f$. Moreover, the right stop mass m_U^2 becomes itself independent of m_0^2 in this limit.

One remark is in order. As we said above, the explicit dependence on A_0 vanishes as the top quark Yukawa coupling approaches its infrared quasi fixed point, and it is replaced by a dependence on the parameter δ . However, as we discussed above, we can only make a reliable perturbative analysis of solutions for which the top quark Yukawa coupling is very close to, but not exactly at, its infrared quasi fixed point value. For these solutions, the explicit dependence on A_0 of the mass parameters is negligible, within a certain range of values for A_0 , which depends on how close to one is the ratio Y_t/Y_f . For instance, if Y_t is at most about ten (one) per cent away from its quasi fixed point value, the dependence on A_0 is negligible for A_0^2 taking values smaller than one (two) order(s) of magnitude of the value of the soft supersymmetry breaking parameters m_0^2 and $M_{1/2}^2$. For still larger values of A_0 the explicit dependence on this parameter may, in principle, reappear. Very large values of A_0 are, however, restricted by the condition ensuring the absence of a colour breaking minimum at M_{GUT} [22],

$$A_0^2 \leq 3(3m_0^2 + \mu_0^2). \quad (16)$$

It is clear that values of A_0 much larger than m_0 and $M_{1/2}$ are consistent with Eq. (16) only for μ (which is of order μ_0) much larger than the universal scalar and gaugino masses. Actually, as will be discussed below, the condition $\mu \gg m_0, M_{1/2}$ is consistent with the requirement of radiative breaking of the electroweak symmetry for $\tan \beta$ close to one. In the following, we will always assume that for values of $\tan \beta$ close to one, we are sufficiently close to the fixed point so that the explicit dependence of the mass parameter $m_{H_2}^2$ on A_0 may be neglected for any A_0 consistent with Eq. (16). This is naturally the case in the numerical solutions we studied. Moreover, we would like to stress that even for $\tan \beta$ closer to one and/or values of Y_t further away from its fixed point value, there is always an interesting range of values for A_0 where the explicit dependence on this parameter can be neglected (although this dependence may reappear by choosing A_0^2 close to its upper bound, Eq. (16)).

4 Radiative Breaking of $SU(2)_L \times U(1)_Y$

In general, the soft supersymmetry breaking parameter space is subject to experimental and theoretical constraints. The experimental constraints come from the present lower bounds on the supersymmetric particle masses. For example the present lower bound on the gluino mass implies a lower bound on the soft supersymmetry breaking parameter $M_{1/2}$ ¹. On the other hand, one should require the stability of the effective potential and a proper breaking of the $SU(2)_L \times U(1)_Y$ symmetry. There are other theoretical constraints, for example those coming from the degree of fine tuning of a given solution. Although a rigorous definition of this concept is lacking, different numerical ways of measuring the degree of fine tuning have been proposed in the literature. In general, independent parameters are assumed. If there were, however, some interrelation between different parameters coming from the fundamental dynamics leading to the soft supersymmetry breaking terms, it would show up in the form of strong correlations between these parameters in the radiative breaking solutions. Hence, if strong correlations are found, the naive fine tuning criteria may be inappropriate for the analysis of the degree of fine tuning of a given solution.

In this work, we have performed a complete numerical analysis of the constraints

¹ The above holds only in the case when one ignores the possibility of a light gluino window. However, the light gluino scenario is ruled out when asking for radiative breaking of $SU(2)_L \times U(1)_Y$ for the infrared fixed point solution, unless one is willing to relax the condition of universality of the soft supersymmetry breaking scalar masses at the unification scale [23], [24].

coming from the requirement of a proper radiative breaking of $SU(2)_L \times U(1)_Y$, the results of which are shown in Figs. 1 to 5. In order to get an analytical understanding of the properties derived numerically, it is most useful to present an approximate theoretical analysis of the radiative breaking condition. From the expressions for the mass parameters obtained above in Eqs. (10)-(14), together with the minimization condition in Eq. (4), and ignoring at this level the radiative corrections to the quartic couplings, it follows that

$$\mu^2 + \frac{M_Z^2}{2} = m_0^2 \frac{1 + 0.5 \tan^2 \beta}{\tan^2 \beta - 1} + M_{1/2}^2 \frac{0.5 + 3.5 \tan^2 \beta}{\tan^2 \beta - 1}. \quad (17)$$

Thus, as we mentioned in the previous section, in the limit $\tan \beta \rightarrow 1$, we find $\mu^2 \gg m_0^2, M_{1/2}^2$. In addition, the above implies that, for a fixed value of $\tan \beta$ and $M_{1/2} > M_Z, m_0$, there is a strong correlation between μ and $M_{1/2}$, which is approximately given by

$$\mu^2 \simeq M_{1/2}^2 \frac{0.5 + 3.5 \tan^2 \beta}{\tan^2 \beta - 1}. \quad (18)$$

If, instead, $m_0^2 \gg M_{1/2}^2, M_Z^2$, a linear correlation between m_0 and μ is obtained,

$$\mu^2 \simeq m_0^2 \frac{1 + 0.5 \tan^2 \beta}{\tan^2 \beta - 1}. \quad (19)$$

The correlations are stronger, for lower values of $\tan \beta$, and become almost exact for $\tan \beta \rightarrow 1$. The inclusion of the radiative corrections to the Higgs quartic couplings give corrections to the effective mass parameters squared, which are of order M_Z^2 and hence do not modify the above behaviour.

The numerical results for the correlations $(m_0 - \mu)$ and $(M_{1/2} - \mu)$, which follow from the requirement of a proper radiative electroweak breaking, are shown in Figs. 1 and 2. We choose values of M_t and $\tan \beta$ for which the top quark Yukawa coupling is one to three per cent away from its infrared quasi fixed point value. In order to fully understand those plots we note that, in the limit $Y_t \rightarrow Y_f$, the following relations hold:

$$m_U^2 \simeq 4M_{1/2}^2, \quad m_0^2 \simeq 2m_Q^2 - 3m_U^2. \quad (20)$$

Thus, as long as we are interested in $SU(2)_L \times U(1)_Y$ breaking, with the common soft supersymmetry breaking parameters such as to give low energy squark masses that are below some common upper bound (which is taken to be 1 TeV in Figs. 1 and 2), then the upper limits for the soft supersymmetry breaking gaugino and scalar masses, $M_{1/2}^U$ and m_0^U , respectively, satisfy the relation $m_0^U \simeq 2\sqrt{3}M_{1/2}^U$. Therefore, in a large portion of the allowed parameter space $m_0^2 \gg M_{1/2}^2$ and, as seen in Fig.1, the correlation $(m_0 - \mu)$

is the dominant, gross feature of the obtained solutions. A closer look at Fig. 1 shows, however, that this correlation is sharper for larger values of m_0 (in particular for low values of $\tan\beta$) and gradually disappears for small m_0 (i.e. in the region where we expect the $(M_{1/2} - \mu)$ correlation predicted for the regime $M_{1/2} > m_0$). The correlation between μ and $M_{1/2}$ is, instead, not explicit in Fig. 2. Again, this is just a reflection of the fact that most of the solutions in Figs. 1 and 2 satisfy the relation $m_0 > M_{1/2}$.

The correlation $(M_{1/2} - \mu)$ becomes sharply visible in the subset of solutions with $M_{1/2} > m_0$. It is very interesting that for $\tan\beta \leq 2$ such solutions (and only those) are selected by the physical requirement of the acceptable neutralino relic abundance, $\Omega h^2 \leq 1$. It is well known that in the minimal supergravity model the neutralino relic abundance is precisely calculable, with no additional free parameters. In Fig. 3 we show the solutions to the electroweak radiative breaking, for $\tan\beta = 1.2$ and $M_t = 160$ GeV, which satisfy the constraint $0.1 \leq \Omega \leq 0.7$. The calculation is based on the formulation of Ref. [25]. First, we compare the available range in $M_{1/2}$ and m_0 without and with the cut on Ωh^2 and observe that the Ω cut gives a strong upper limit on m_0 and selects solutions with $M_{1/2} \geq m_0$. This upper bound on m_0 has a very simple qualitative explanation. The low $\tan\beta$ solutions to radiative breaking always give $\mu > M_{1/2}$ (as follows from Eq. (18) and is seen in Fig. 2) and, consequently, the predicted lightest neutralinos are strongly bino-like. Their annihilation then proceeds mainly via slepton exchange and the requirement of the acceptable relic abundance (which is inversely proportional to the annihilation cross section) puts an upper bound on the slepton mass, i.e., also on the grand unification parameter m_0 . Then, the other features seen in Fig. 3 follow naturally: strong $(M_{1/2} - \mu)$ correlation and no $(m_0 - \mu)$ correlation.

The second radiative breaking condition, Eq. (3), leads to the following relation

$$\sin 2\beta \left(2\mu^2 + \frac{m_0^2}{2} - 3M_{1/2}^2 \right) = 2\mu (\delta + 0.6M_{1/2}). \quad (21)$$

Additional properties of the solution may be obtained by using Eq. (21). As we mentioned above, we define the mass parameters m_3^2 , m_0 and $M_{1/2}$ to be positive. With this sign convention, there are two different regimes, depending on the sign of the supersymmetric mass parameter μ . Since, for example, in the region $M_{1/2} > M_Z, m_0$ there is a strong correlation, Eq.(18), between μ and $M_{1/2}$, Eq. (21) shows that for $\mu > 0$ there is a strong linear correlation between the parameters δ and $M_{1/2}$,

$$\delta \simeq M_{1/2} \left(-0.6 + \frac{2 \sin 2\beta (1 + \tan^2 \beta)}{\sqrt{(0.5 + 3.5 \tan^2 \beta) (\tan^2 \beta - 1)}} \right), \quad (22)$$

and also between μ and δ ,

$$\delta = \mu \frac{\left(-0.6\sqrt{(0.5 + 3.5 \tan^2 \beta)(\tan^2 \beta - 1)} + 2 \sin(2\beta)(1 + \tan^2 \beta)\right)}{0.5 + 3.5 \tan^2 \beta}. \quad (23)$$

Analogously, in the region $m_0 \gg M_{1/2}$, where the linear correlation between m_0 and μ holds, we obtain a strong linear correlation between δ and m_0

$$\delta \simeq m_0 \frac{\sin(2\beta)0.75(1 + \tan^2 \beta)}{\sqrt{(1 + 0.5 \tan^2 \beta)(\tan^2 \beta - 1)}}, \quad (24)$$

as well as a different correlation between μ and δ ,

$$\delta = \mu \frac{\sin(2\beta)0.75(1 + \tan^2 \beta)}{1 + 0.5 \tan^2 \beta}. \quad (25)$$

For $\mu \leq 0$, instead, the correlation between δ and $M_{1/2}$ in the $M_{1/2} > m_0, M_Z$ regime reads

$$\delta \simeq -M_{1/2} \left(0.6 + \frac{2 \sin 2\beta (1 + \tan^2 \beta)}{\sqrt{(0.5 + 3.5 \tan^2 \beta)(\tan^2 \beta - 1)}}\right). \quad (26)$$

It is interesting to compare Eqs. (22) and (26). A variation in the sign of μ yields a different absolute value of the coefficients relating δ to $M_{1/2}$. Hence, the resulting correlations are not symmetric under a change in sign of δ . The linear correlation between δ and μ in this regime is hence given by

$$\delta \simeq \mu \frac{\left(0.6\sqrt{(0.5 + 3.5 \tan^2 \beta)(\tan^2 \beta - 1)} + 2 \sin(2\beta)(1 + \tan^2 \beta)\right)}{0.5 + 3.5 \tan^2 \beta}. \quad (27)$$

Furthermore, in the region $m_0^2 \gg M_{1/2}^2$, the following linear correlation between δ and m_0 is present for $\mu \leq 0$,

$$\delta \simeq -m_0 \frac{\sin(2\beta)0.75(1 + \tan^2 \beta)}{\sqrt{(1 + 0.5 \tan^2 \beta)(\tan^2 \beta - 1)}}. \quad (28)$$

The above expression differs only in sign from the one obtained in Eq. (24), for the regime $\mu \geq 0$, while the resulting correlation between μ and δ is exactly that obtained in Eq. (25).

Interestingly enough, although they have quite a different dependence on $\tan \beta$, for $\mu \leq 0$, the numerical values of the coefficients relating δ with μ in the two different regimes studied above, Eqs. (25) and (27), are remarkably close to each other for $\tan \beta \leq 10$. Hence, for those values of $\tan \beta$ a strong correlation between δ and μ is expected to

appear, for $\mu \leq 0$, for the whole range of values of m_0 and $M_{1/2}$. Due to the numerical value of the coefficients, the correlation between μ and δ in the regime $\mu \leq 0$ should improve for $\tan \beta$ close to 1 as well as for $\tan \beta$ close to 5. This is actually observed in Fig. 4.

In the region $\mu \geq 0$, instead, the numerical values of the two coefficients relating μ and δ only coincide in the limit $\tan \beta \rightarrow 1$, becoming quite different as $\tan \beta$ increases. Hence, for $\mu \geq 0$ we expect the correlation $(\mu - \delta)$ to be good only for small values of $\tan \beta \simeq 1$. On the contrary, for larger values of $\tan \beta$ this linear correlation is lost. This is due to the fact that the coefficients in both regimes are quite different and, in addition, the correlation of μ with $M_{1/2}$ and m_0 in the two studied regimes becomes weaker than for lower values of $\tan \beta$. For instance, for $\tan \beta = 10$ the correlations observed for smaller values of $\tan \beta$ almost disappear, as is shown in Fig. 4.

A very similar analysis applies to the correlation between the pseudoscalar mass m_A and the parameter δ . Combining Eqs. (10) and (11) with Eqs. (22) and (23) (or (26) and (28)) it is easy to find that, in the regime $M_{1/2} > m_0$, the linear relation

$$m_A = -\delta \frac{\sqrt{2(1 + 7 \tan^2 \beta)(1 + \tan^2 \beta)}}{0.6\sqrt{(0.5 + 3.5 \tan^2 \beta)(\tan^2 \beta - 1)} + (-)4 \tan \beta} \quad (29)$$

appears for $\mu < 0$ ($\mu > 0$), while for $m_0^2 \gg M_{1/2}^2$ one finds

$$m_A = -(+)\delta \sqrt{\frac{3(1 + \tan^2 \beta)(2 + \tan^2 \beta)}{9 \tan^2 \beta}}. \quad (30)$$

The properties of the correlation between m_A and δ are similar to those ones of the $(\delta - \mu)$ correlation. Indeed, in the limit $\tan \beta \rightarrow 1$ there exists a strong linear correlation between m_A and δ , in both regimes, $M_{1/2} > M_Z, m_0$ and $m_0 \gg M_{1/2}$. This is clearly seen in the numerical results shown in Fig. 5. As before, this correlation is stable for small and moderate values of $\tan \beta$ in the $\mu < 0$ regime, while it rapidly disappears in the $\mu > 0$ regime when $\tan \beta$ becomes larger than 1. In general, the correlation becomes weaker for increasing values of $\tan \beta$.

One remark should be made regarding the condition $\delta = 0$. Since at the infrared quasi fixed point the explicit dependence on the A_0 parameter is replaced by the dependence on δ , finding solutions that satisfy the relation $\delta = 0$ would also imply to have solutions for $A_0 = B_0 = 0$. The condition $\delta = 0$ cannot be fulfilled for values of $\mu \leq 0$, for $\tan \beta < 10$, since the strong correlation between μ and δ renders this impossible. For $\mu > 0$ and

moderate values of $\tan\beta$, where the correlation between δ and μ is lost, this condition may be achieved. Indeed, the complete study of the minimization condition shows that $\delta = 0$ is achievable for $\tan\beta \geq 4$ for positive values of the mass parameter μ .

5 Conclusions

In this article, we have studied the properties of the minimal supergravity model for the case in which the top quark mass is close to its infrared quasi fixed point solution. To study the regime of the infrared quasi fixed point solution for m_t is of interest for various reasons. This solution explains in a natural way the relatively large value of the top quark mass. Moreover, it appears as a prediction in many interesting theoretical scenarios. In particular, it has been shown that for the presently allowed values of the bottom quark mass and the electroweak gauge couplings, the conditions of gauge and bottom-tau Yukawa coupling unification imply a strong convergence of the top quark mass to its infrared fixed point value.

Our main conclusion is that a proximity of the top quark mass to its infrared quasi fixed point values would have important implications for the mechanism of radiative electroweak symmetry breaking in the minimal supergravity model. In particular, we show that there is a reduction in the number of independent parameters: the low energy parameters do not depend on the grand unification scale parameters A_0 and B_0 , but on the combination $\delta = B_0 - A_0/2$. Furthermore, we prove the existence of important correlations between the remaining free parameters of the model, which emerge as the exact pattern of all solutions with proper electroweak symmetry breaking. These correlations become particularly strong in the low $\tan\beta$ region which corresponds to the infrared fixed point values of the top quark mass $M_t \leq 180$ GeV. It is tempting to speculate that they have some fundamental explanation. The usual criteria of fine tuning, with all parameters treated as independent, would then have to be abandoned.

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FIGURE CAPTIONS

Fig. 1. Solutions for the parameters m_0 and μ obtained by scanning the parameter space with the requirement of a proper radiative electroweak symmetry breaking solution, for four different values of M_t and $\tan\beta$ consistent with the infrared quasi fixed point solution. The scanning of solutions was performed considering values of m_A , m_Q and m_U up to 1 TeV.

Fig. 2. Same as in Fig. 1, but for the radiative electroweak symmetry breaking solutions in the $M_{1/2} - \mu$ parameter space.

Fig. 3. Analysis of the Ω cut effects in the radiative electroweak symmetry breaking solutions in the mass parameter space, for a given set of values of M_t and $\tan\beta$ consistent with the infrared quasi fixed point solution.

Fig. 4. Same as in Fig. 1, but for the radiative electroweak symmetry breaking solutions in the $\mu - \delta$ parameter space.

Fig. 5. Same as in Fig. 1, but for the radiative electroweak symmetry breaking solutions in the $m_A - \delta$ parameter space.

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